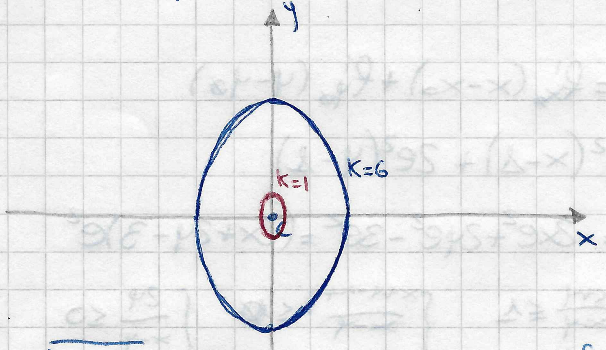
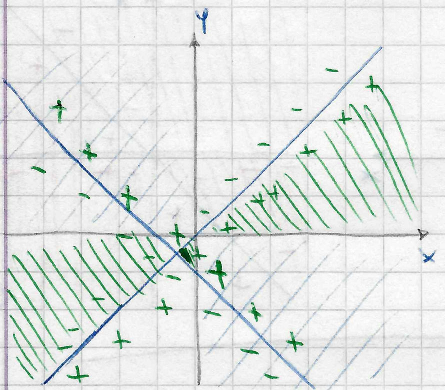
$z = \sqrt{9x^2+4y^2}$ $\qquad 9x^2+4y^2 = K^2$ $\qquad$ elisse concentriche con centro $C(\varnothing;\varnothing)$



x	y
$\varnothing$	$\pm\frac{K}{2}$
$\pm\frac{K}{3}$	$\varnothing$

• $z = \sqrt{\dfrac{x+y+1}{x-y}} + \sqrt{xy}$ $\qquad$ DOMINIO $\begin{cases} xy \geq \varnothing \\ \dfrac{x+y+1}{x-y} \geq 0 \end{cases}$ $\begin{cases} \text{tutti i punti del I° e III° quad} \\ N \geq 0 \quad y \geq -x-1 \\ D > 0 \quad y < x \end{cases}$



$$z'_x = \frac{1}{2\sqrt{}} \cdot \frac{x-y-\cancel{x}-y-1}{(x-y)^2} + \frac{1}{2\sqrt{xy}} \cdot y =$$

$$= \frac{1}{2\sqrt{}} \cdot \frac{-2y-1}{(x-y)^2} + \frac{y}{2\sqrt{xy}}$$

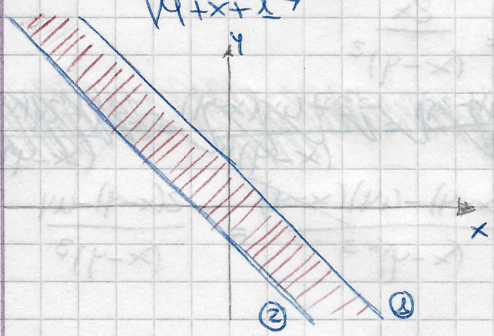
$$z'_y = \frac{1}{2\sqrt{}} \cdot \frac{x-y+x+y+1}{(x-y)^2} + \frac{1}{2\sqrt{xy}} \cdot x =$$

$$= \frac{1}{2\sqrt{}} \cdot \frac{2x+1}{(x-y)^2} + \frac{x}{2\sqrt{xy}}$$

piano tangente in $P(2;1;2+\sqrt{2})$

$f'_x(2;1) = -\frac{3}{4} + \frac{1}{2\sqrt{2}} = \frac{-3\sqrt{2}+2}{4\sqrt{2}}$ $\qquad f'_y = +\frac{5}{4} + \frac{1}{\sqrt{2}} = \frac{5\sqrt{2}+4}{4\sqrt{2}}$

• $z = \dfrac{\ln(-x-y+1)}{\sqrt{y+x+1}}$ $\qquad$ DOMINIO $\begin{cases} -x-y+1 > 0 \\ y+x+1 > 0 \end{cases}$ $\begin{cases} y < -x+1 & ① \\ y > -x-1 & ② \end{cases}$



• $z = \operatorname{sen} xy \cdot \cos 2x$

$z'_x = +y\cos(xy)(\cos 2x) + \operatorname{sen}(xy)(-2\operatorname{sen} 2x)$
$= y\cos(xy)\cdot\cos 2x - 2\operatorname{sen}(xy)\operatorname{sen} 2x$

$z'_y = x\cos(xy)\cos 2x + \varnothing =$
$= x\cos xy(\cos 2x)$

• $z = \operatorname{arctg}(x^2+y^2)$

$z'_x = \dfrac{2x}{(x^2+y^2)^2+1}$ $\qquad z'_y = \dfrac{2y}{(x^2+y^2)^2+1}$

• $z = 5x^2 + 3x^4y^5 - 5 + 3y$

$z'_x = 10x + 12x^3y^5$ $\qquad z'_y = 15x^4y^4 + 3$

• $z = e^{x^2+y^2}$ piani tangenti in $P(1,1,e^2)$

$z'_x = 2xe^{x^2+y^2}$ $\quad$ $z'_y = 2ye^{x^2+y^2}$

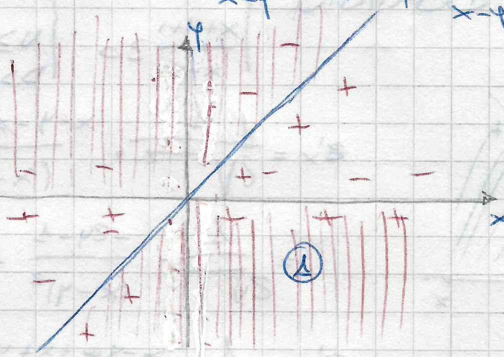
$f'_{x_0} = 2e^2$ $\quad$ $f'_{y_0} = 2e^2$ $\quad$ $z - z_0 = f'_{x_0}(x-x_0) + f'_{y_0}(y-y_0)$

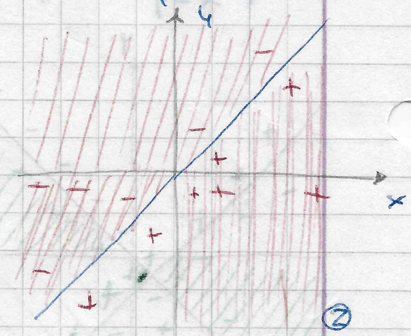
$$z - e^2 = 2e^2(x-1) + 2e^2(y-1)$$

$\to z = 2xe^2 - 2e^2 + 2ye^2 - 2e^2 + e^2 = 2xe^2 + 2ye^2 - 3e^2 = (2x+2y-3)e^2$

• $z = \arcsin\frac{x+y}{x-y}$ $\quad$ DOMINIO $\begin{cases}\frac{x+y}{x-y} \le 1 \\ \frac{x+y}{x-y} \ge -1\end{cases}$ $\begin{cases}\frac{x+y-x+y}{x-y} \le 0 \\ \frac{x+y+x-y}{x-y} \ge 0\end{cases}$ $\begin{cases}\frac{2y}{x-y} \le 0 \\ \frac{2x}{x-y} \ge 0\end{cases}$

$\begin{cases}-2y \ge 0 \\ x-y > 0 \\ 2x \ge 0 \\ x-y > 0\end{cases}$ $\begin{cases}y \le 0 \\ y < x \\ x \ge 0 \\ y < x\end{cases}$ $\begin{matrix}① \\ \\ ②\end{matrix}$





$z = x^3y^2 - 5x^2y^4$

$z'_x = 3x^2y^2 - 10xy^4$ $\quad$ $z''_{xx} = 6xy^2 - 10y^4$

$z''_{xy} = 6x^2y - 40xy^3$

$z''_{yx} = 6x^2y - 40xy^3$

$z'_y = 2x^3y - 20x^2y^3$ $\quad$ $z''_{yy} = 2x^3 - 60x^2y^2$

Per il PRINCIPIO DI SCHWARZ, le derivate miste sono sempre uguali!

• $z = \frac{x+y}{x-y}$ $\quad$ DOMINIO $(x \neq y)$

$z'_x = \frac{x-y-x-y}{(x-y)^2} = \frac{-2y}{(x-y)^2}$ $\quad$ $z'_y = \frac{x-y+x+y}{(x-y)^2} = \frac{2x}{(x-y)^2}$

$z''_{xx} = \frac{+4y(x-y)}{(x-y)^4} = \frac{+4y\cancel{(x-y)}}{(x-y)^{\cancel{4}\,3}}$

$z''_{xx} = \frac{4y}{(x-y)^3}$ $\quad$ $z''_{xy} = \frac{-2(x-y)^2 - 4y(x-y)}{(x-y)^4} = \frac{(-2(x-y)-4y)\cancel{(x-y)}}{(x-y)^{\cancel{4}\,3}} = \frac{-2(x-y)-4y}{(x-y)^3}$

$z''_{xy} = \frac{-2x-2y}{(x-y)^3}$

$z''_{yy} = \frac{4x}{(x-y)^3}$ $\quad$ $z''_{yx} = \frac{2(x-y)^2 - 2(x-y)2x}{(x-y)^4} = \frac{2x-2y-4x}{(x-y)^3} = \frac{-2x-2y}{(x-y)^3}$

$z = \sin x \sin y$ $\quad$ $z'_x = \cos x \sin y$ $\quad$ $z''_{xx} = -\sin x \sin y$

$z''_{xy} = \cos x \cos y$

$z'_y = \sin x \cos y$ $\quad$ $z''_{yx} = \cos x \cos y$

$z''_{yy} = -\sin x \sin y$